PPC Energy Momentum Tensor Equations (Perfect Fluid Form) and PPC weak field Equations

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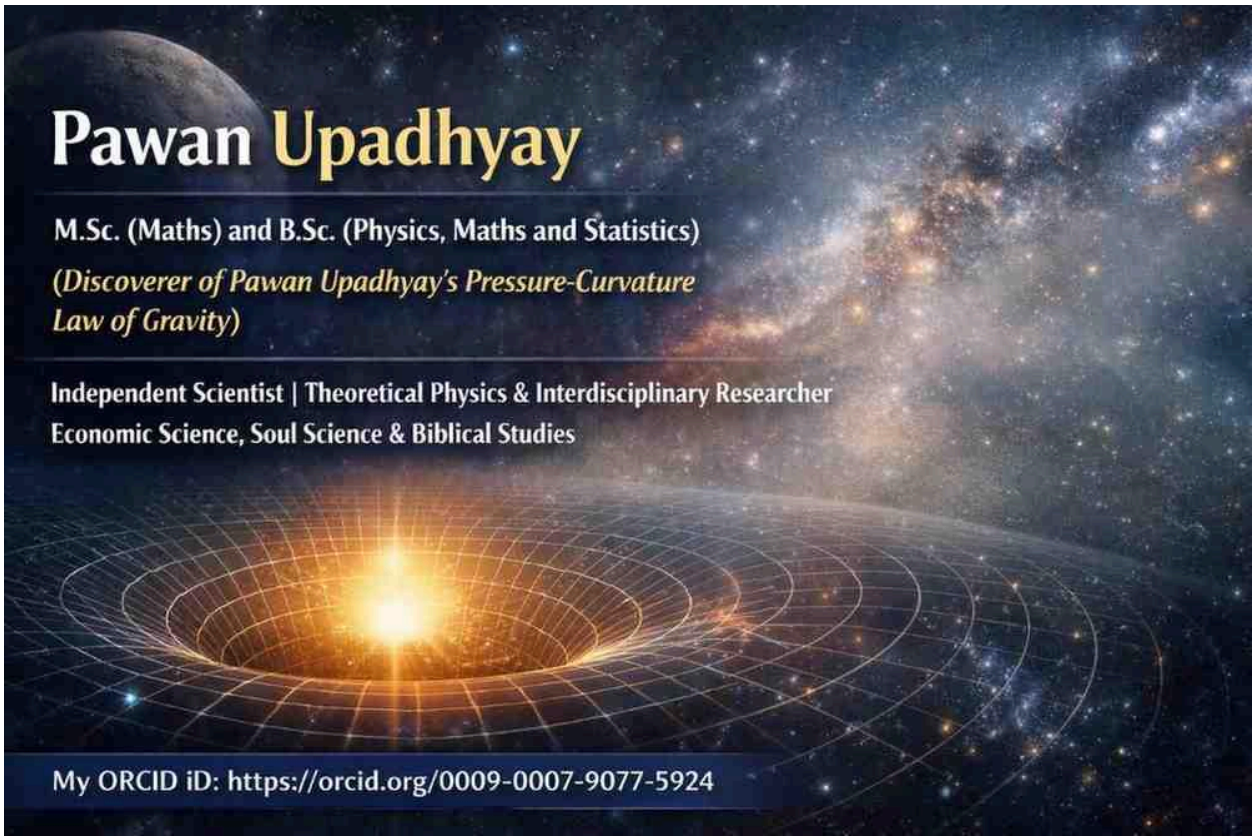
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Equation of State

$$P_g = \omega E_d$$

Energy–Momentum Tensor (Perfect Fluid Form)

$$T_{\mu\nu} = (E_d + P_g)\, u_\mu u_\nu + P_g\, g_{\mu\nu}$$

Substitute $P_g = \omega E_d$

$$T_{\mu\nu} = (E_d + \omega E_d)\, u_\mu u_\nu + \omega E_d\, g_{\mu\nu}$$

$$T_{\mu\nu} = E_d(1+\omega)u_\mu u_\nu + \omega E_d g_{\mu\nu}$$

Factor E_d:

$$T_{\mu\nu} = E_d\left[(1+\omega)u_\mu u_\nu + \omega g_{\mu\nu}\right]$$

Einstein Field Equation

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Substitute $T_{\mu\nu}$:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} E_d \left[(1+\omega) u_\mu u_\nu + \omega g_{\mu\nu}\right]$$

Special Cases

Dust (Matter)

$$\omega = 0$$

$$T_{\mu\nu} = E_d u_\mu u_\nu$$

$$G_{\mu\nu} = \frac{8\pi G}{c^4} E_d u_\mu u_\nu$$

Radiation

$$\omega = \frac{1}{3}$$

$$T_{\mu\nu} = E_d \left[\frac{4}{3} u_\mu u_\nu + \frac{1}{3} g_{\mu\nu}\right]$$

$$G_{\mu\nu} = \frac{8\pi G}{c^4} E_d \left[\frac{4}{3} u_\mu u_\nu + \frac{1}{3} g_{\mu\nu}\right]$$

Stiff Matter

$$\omega = 1$$

$$T_{\mu\nu} = E_d \left[2u_\mu u_\nu + g_{\mu\nu}\right.$$

$$G_{\mu\nu} = \frac{8\pi G}{c^4} E_d \left[2u_\mu u_\nu + g_{\mu\nu}\right]$$

Dark Energy

$$\omega = -1$$

$$T_{\mu\nu} = -E_d g_{\mu\nu}$$

$$G_{\mu\nu} = -\frac{8\pi G}{c^4} E_d g_{\mu\nu}$$

Einstein's weak-field limit is the approximation of General Relativity that applies when:

- The gravitational field is weak: $|\Phi|/c^2 \ll 1$
- Objects move much slower than the speed of light: $v \ll c$

In this limit, Einstein's field equations reduce to the familiar Newtonian law of gravity.

The key weak-field equation is:

$$\nabla^2 \Phi = 4\pi G \rho,$$

where:

- Φ = Newtonian gravitational potential,
- G = gravitational constant,
- ρ = mass density.

This is the **Poisson equation**, and it reproduces Newton's law of gravitation.

The spacetime metric in the weak-field approximation becomes:

$$g_{00} \approx -\left(1 + \frac{2\Phi}{c^2}\right),$$

$$g_{ij} \approx \delta_{ij}\left(1 - \frac{2\Phi}{c^2}\right),$$

where:

- $g_{\mu\nu}$ is the spacetime metric,
- δ_{ij} is the Kronecker delta.

The complete Einstein field equations, from which the weak-field limit is derived, are:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu},$$

Therefore:

- **Newtonian gravity:**

$$\nabla^2 \Phi = 4\pi G\rho.$$

- **General relativistic weak-field approximation (static perfect fluid):**

$$\nabla^2 \Phi \approx 4\pi G \left(\rho + \tfrac{3p}{c^2}\right).$$

Energy Density as the Source of Gravity in the Weak-Field Limit

Einstein's mass–energy equivalence,

$$E = mc^2,$$

implies that mass density and energy density are directly related. If E_d denotes the **energy density** (energy per unit volume), then the corresponding mass density is

$$\rho = \frac{E_d}{c^2},$$

where:

- ρ is the mass density ($\mathrm{kg}, \mathrm{m}^{-3}$),
- E_d is the energy density ($\mathrm{J}, \mathrm{m}^{-3}$),
- c is the speed of light.

In the weak-field, slow-motion limit of General Relativity, Einstein's field equations reduce to the Newtonian Poisson equation,

$$\nabla^2 \Phi = 4\pi G \rho,$$

where:

- Φ is the Newtonian gravitational potential,
- G is the gravitational constant.

Substituting the mass–energy relation,

$$\rho = \frac{E_d}{c^2},$$

into the Poisson equation yields

$$\nabla^2 \Phi = \frac{4\pi G}{c^2} E_d.$$

This equation demonstrates that, in the weak-field approximation, the gravitational potential may be expressed directly in terms of energy density. Since mass density is equivalent to energy density divided by c^2, gravity in this regime can be interpreted as being sourced by energy density rather than mass density alone.

This reformulation is mathematically equivalent to the standard Newtonian expression and is entirely consistent with Einstein's mass–energy equivalence. It emphasizes that mass is not an independent gravitational source but rather a manifestation of energy.

It is important to recognize that this equivalence applies specifically to the weak-field, slow-motion limit. In the complete framework of General Relativity, gravity is determined by the full stress-energy tensor T_μv, which includes not only energy density but also momentum density, stresses, and pressure. Therefore, while energy density is the dominant source in weak gravitational fields, pressure and other forms of stress-energy also contribute to spacetime curvature in strong-field or relativistic regimes.

The **Poisson equation for Newtonian gravity** is

$$\boxed{\nabla^2 \Phi = 4\pi G\rho}$$

where:

- $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$ is the Laplacian operator,
- $\Phi(x, y, z)$ is the gravitational potential,
- G is the universal gravitational constant,
- $\rho(x, y, z)$ is the mass density.

Expanding the Laplacian explicitly, the full Poisson equation is

$$\boxed{\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 4\pi G\rho}$$

Using Einstein's mass–energy relation,

$$\rho = \frac{E_d}{c^2},$$

the equation can be written in terms of **energy density**:

$$\boxed{\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = \frac{4\pi G}{c^2} E_d}$$

or, in compact notation,

$$\boxed{\nabla^2 \Phi = \frac{4\pi G}{c^2} E_d.}$$

This is the weak-field form of Einstein's theory expressed directly in terms of **energy density**, where E_d is the local energy per unit volume. In full General Relativity, however, the source of gravity is the complete stress-energy tensor $T_{\mu\nu}$, not energy density alone.

Pressure as a Source of Gravity in the Relativistic Poisson Equation

In Newtonian gravity, the gravitational potential is determined solely by the mass density of matter. The Poisson equation is

$$\nabla^2 \Phi = 4\pi G \rho,$$

where Φ is the gravitational potential, G is the gravitational constant, and ρ is the mass density.

General Relativity extends this concept by recognizing that gravity is produced not only by mass but by the entire stress-energy content of matter. In addition to energy density, the pressure within matter also contributes to the gravitational field. For a static, isotropic perfect fluid, the relativistic Poisson equation is

$$\nabla^2\Phi = 4\pi G\left(\rho + \frac{3p}{c^2}\right),$$

where:

- ∇^2 is the Laplacian operator,
- Φ is the gravitational potential,
- ρ is the mass density,
- p is the isotropic pressure,
- c is the speed of light.

Physical Meaning of the Equation

The first term, ρ, represents the familiar contribution of mass density to gravity. The second term,

$$\frac{3p}{c^2},$$

is the relativistic contribution arising from pressure.

The factor of **3** appears because pressure acts independently along the three spatial directions (x, y, and z). In the stress-energy tensor of a perfect fluid,

$$T_{\mu\nu} = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix},$$

each spatial pressure component contributes equally to spacetime curvature. Summing these three equal contributions produces the total pressure term $3p$.

Expression in Terms of Energy Density

Using Einstein's mass-energy relation,

$$\rho = \frac{E_d}{c^2},$$

where E_d is the energy density, the relativistic Poisson equation becomes

$$\boxed{\nabla^2 \Phi = \frac{4\pi G}{c^2} \left(E_d + 3p \right).}$$

This form emphasizes that the effective gravitational source consists of both **energy density** and **pressure**.

Physical Interpretation

This equation demonstrates one of the central insights of General Relativity: gravity is generated not only by mass but by the full energy content of matter, including its internal pressure. In ordinary conditions, such as within planets or everyday materials, the pressure term is much smaller than the energy-density term and can usually be neglected. However, in extremely dense astrophysical objects—such as neutron stars, collapsing stellar cores, and the early universe—pressure becomes comparable to the energy density and contributes significantly to spacetime curvature.

In the PPC Law of Gravity, the pressure term is represented by the gravitational pressure P_g, which denotes the effective pressure responsible for contributing to spacetime curvature within the PPC framework. While General Relativity expresses the relativistic correction using the thermodynamic pressure p, the PPC framework uses P_g as its fundamental gravitational pressure variable.

$$\nabla^2\Phi = 4\pi G\left(\rho + \frac{3P_g}{c^2}\right),$$

or, using energy density,

$$\nabla^2\Phi = \frac{4\pi G}{c^2}\left(E_d + 3P_g\right).$$

In the **PPC Law of Gravity**, the pressure contribution is represented by the **gravitational pressure**, denoted by P_g. Gravitational pressure is defined as the effective pressure associated with the local concentration of energy that contributes to spacetime curvature. Accordingly, the PPC gravitational field equation is written as

$$\boxed{\nabla^2 \Phi = 4\pi G \left(\rho + \frac{3P_g}{c^2} \right).}$$

Using Einstein's mass–energy equivalence,

$$\rho = \frac{E_d}{c^2},$$

where E_d is the local energy density, the PPC equation becomes

$$\boxed{\nabla^2 \Phi = \frac{4\pi G}{c^2} \left(E_d + 3P_g \right).}$$

This equation shows that the gravitational source consists of two fundamental contributions: **energy density** and **gravitational pressure**. The energy density E_d represents the amount of energy contained within a unit volume, while the gravitational pressure P_g represents the effective pressure generated by that concentrated energy. Together, these quantities determine the curvature of spacetime and the resulting gravitational field.

The factor of **3** arises because pressure contributes independently along each of the three spatial dimensions (x, y, and z). Since the pressure is isotropic, the total pressure contribution is the sum of these three equal components, producing the term $3P_g$.

Unlike the traditional Newtonian interpretation, which attributes gravity solely to mass density, the PPC Law of Gravity emphasizes that gravity is governed by both the **amount of energy present** and the **gravitational pressure associated with its concentration**. As the local energy density becomes more concentrated, the corresponding gravitational pressure increases, leading to stronger spacetime curvature. Consequently, dense matter produces a stronger gravitational field than diffuse matter of the same total mass because higher energy density generates greater gravitational pressure.

The effective gravitational source in the PPC framework can therefore be written as

$$\rho_{\text{eff}} = \rho + \frac{3P_g}{c^2} = \frac{E_d + 3P_g}{c^2}.$$

This formulation provides a physical interpretation of gravity in which **energy density and gravitational pressure jointly determine the strength of spacetime curvature**, while remaining consistent with Einstein's principle that gravity is fundamentally sourced by energy rather than mass alone.

$$\boxed{P_g = \omega E_d,}$$

where:

- P_g = gravitational pressure,
- E_d = local energy density,
- ω = dimensionless equation-of-state parameter.

Substituting this relation into the PPC gravitational field equation,

$$\nabla^2 \Phi = \frac{4\pi G}{c^2} \left(E_d + 3P_g \right),$$

gives

$$\nabla^2 \Phi = \frac{4\pi G}{c^2} \left(E_d + 3\omega E_d \right),$$

or equivalently,

$$\boxed{\nabla^2 \Phi = \frac{4\pi G}{c^2}(1+3\omega)E_d.}$$

Interpretation

In the PPC Law of Gravity, the gravitational pressure is assumed to obey the equation of state

$$P_g = \omega E_d,$$

where the dimensionless parameter ω specifies the ratio of gravitational pressure to local energy density. This relation is analogous in form to the equation of state commonly used in relativistic cosmology, but within the PPC framework it defines the **gravitational pressure** associated with the local energy distribution.

The effective gravitational source is therefore

$$\boxed{\rho_{\text{eff}} = \frac{(1+3\omega)E_d}{c^2}.}$$

The parameter ω determines how strongly gravitational pressure enhances the gravitational field beyond the contribution of energy density alone. When $\omega = 0$, the gravitational pressure vanishes and the equation reduces to the standard weak-field Poisson equation expressed in terms of energy density. As ω increases, the gravitational pressure contributes more significantly to spacetime curvature through the factor $(1+3\omega)$.

In the PPC framework, ω is therefore a **dimensionless equation-of-state parameter** that characterizes the relationship between gravitational pressure and energy density, while the combination $E_d + 3P_g$ serves as the effective source of the gravitational field.

Using

$$P_g = \omega E_d,$$

the PPC field equation becomes

$$\boxed{\nabla^2 \Phi = \frac{4\pi G}{c^2}(1 + 3\omega) E_d.}$$

Below are the corresponding equations for commonly used values of the equation-of-state parameter ω:

Equation-of-state parameter ω
$\omega = 0$ (pressureless matter)
$\omega = \frac{1}{3}$ (radiation)
$\omega = 1$ (stiff matter)
$\omega = -\frac{1}{3}$
$\omega = -\frac{2}{3}$
$\omega = -1$ (vacuum energy)

Equation-of-state parameter ω	PPC gravitational equation
$\omega = 0$ (pressureless matter)	$\nabla^2\Phi = \frac{4\pi G}{c^2} E_d$
$\omega = \frac{1}{3}$ (radiation)	$\nabla^2\Phi = \frac{8\pi G}{c^2} E_d$
$\omega = 1$ (stiff matter)	$\nabla^2\Phi = \frac{16\pi G}{c^2} E$
$\omega = -\frac{1}{3}$	$\nabla^2\Phi = 0$
$\omega = -\frac{2}{3}$	$\nabla^2\Phi = -\frac{4\pi G}{c^2} l$
$\omega = -1$ (vacuum energy)	$\nabla^2\Phi = -\frac{8\pi G}{c^2} l$

Full Image of PPC gravitational field equation.

PPC gravitational field equation	Effective source
$\nabla^2\Phi = \frac{4\pi G}{c^2} E_d$	$\rho_{\text{eff}} = \frac{E_d}{c^2}$
$\nabla^2\Phi = \frac{8\pi G}{c^2} E_d$	$\rho_{\text{eff}} = \frac{2E_d}{c^2}$
$\nabla^2\Phi = \frac{16\pi G}{c^2} E_d$	$\rho_{\text{eff}} = \frac{4E_d}{c^2}$
$\nabla^2\Phi = 0$	$\rho_{\text{eff}} = 0$
$\nabla^2\Phi = -\frac{4\pi G}{c^2} E_d$	$\rho_{\text{eff}} = -\frac{E_d}{c^2}$
$\nabla^2\Phi = -\frac{8\pi G}{c^2} E_d$	$\rho_{\text{eff}} = -\frac{2E_d}{c^2}$

Interpretation

The PPC field equation predicts that the effective gravitational source is

$$\boxed{\rho_{\text{eff}} = \frac{(1+3\omega)E_d}{c^2}.}$$

The factor $(1 + 3\omega)$ determines how gravitational pressure modifies the strength of the gravitational field:

- $\omega = 0$: Gravity is produced only by energy density.
- $\omega > 0$: Gravitational pressure strengthens the gravitational field.
- $\omega = -\frac{1}{3}$: The pressure contribution exactly cancels the energy-density contribution, giving zero effective source in this equation.
- $\omega < -\frac{1}{3}$: The effective source becomes negative, corresponding to a repulsive gravitational contribution within this formulation.

For most physics papers, however, it is common and acceptable to write simply

$$\boxed{P_g(x^\mu) = \omega(x^\mu) E_d(x^\mu), \qquad x^\mu \in M,}$$

where $x^\mu = (ct, x, y, z)$ denotes the spacetime coordinates of a point in the manifold.

> **The pressure forces determine both the magnitude and the shape of spacetime curvature. Specifically, $E_d + P_g$ determines the magnitude of the curvature, while ∇P_g determines the shape of the curvature and, consequently, the trajectory (orbit) of matter.**

> **Pressure forces determine the magnitude and the shape of spacetime curvature: $E_d + P_g$ governs the magnitude of curvature, whereas ∇P_g governs its shape and the resulting orbital motion.**

This wording clearly distinguishes the two roles:

- $E_d + P_g \rightarrow$ controls the **strength (magnitude)** of curvature.
- $\nabla P_g \rightarrow$ controls the **geometry (shape)** of curvature, thereby determining the path or orbit followed by matter.

In Weak field limit (Static perfect fluid),

- (E_d+3P_g) controls the strength (magnitude) of curvature.
- Pressure gradient (∇P_g) controls the geometry (shape) of curvature, thereby determining the path or orbit followed by matter.

- **Magnitude of curvature:** $E_d + 3P_g$
- **Shape (geometry) of curvature:** ∇P_g